Mecanum Wheel Odometry

FTC 9866 Virus

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Introduction

Odometry requires three non-powered omni wheels connected to encoders: two forward facing wheels and one lateral wheel located at the back of the robot.

To simplify this task, we can make a few assumptions, the first of which is making motor powers constant. Since we update odometry once a loop, the same frequency at which we update motor powers, the motor powers won't change between two consecutive odometry calls. As a result, we have only the 5 different movements that are possible with constant motor powers to account for in the algorithm. These are:

- Forwards/backwards line

- Strafing line

- Diagonal line

- Forwards/backwards arc

- Strafing arc

We can also treat lines as if they are arcs with infinite radius. As a result, we can combine the forwards/backwards line and left/right line into their respective arcs. This drops the number of cases to three:

- Diagonal line

- Forwards/backwards arc

- Strafing arc

One thing to notice is that with the forwards/backwards arc, the robot's front is always facing the direction of movement, while with the strafing arc, it's instead the robot's side that's facing the direction of movement. What this means is that the forwards/backwards arc only has a forwards/backwards component to it, while the strafing arc only has a strafing component to it. The diagonal line can be split into both forwards/backwards and strafing components, similar to how vectors can be split into x and y components. Because of this, we can solve for the forwards/backwards and strafing components separately, and then add them together at the end.

Forwards/Backwards Movement
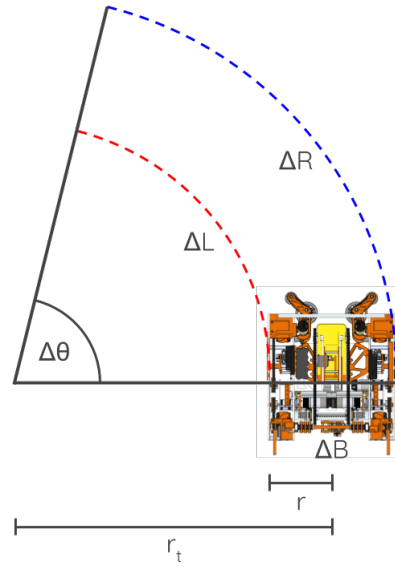
The first task is to find the change in heading over the course of the loop iteration. This can be found by treating the movement as an arc and solving for the angle.

$$\Delta L = (r_t - r)\Delta\theta$$

$$\Delta R = (r_t + r)\Delta\theta$$

Subtracting the first equation from the second yields

$$\Delta R - \Delta L = 2r\Delta\theta$$

$$\Delta\theta = \frac{\Delta R - \Delta L}{2r}$$

Now we must solve for the turning radius, using the original arc equations.

$$\Delta\theta = \frac{\Delta L}{r_t - r}$$

$$\Delta\theta = \frac{\Delta R}{r_t + r}$$

$$\frac{\Delta L}{r_t - r} = \frac{\Delta R}{r_t + r}$$

$$\Delta L r_t + \Delta L r = \Delta R r_t - \Delta R r$$

$$\Delta L r_t - \Delta R r_t = -\Delta L r - \Delta R r$$

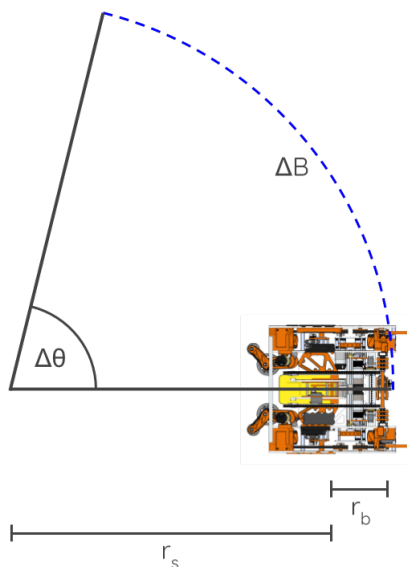$$r_t(\Delta L - \Delta R) = -r(\Delta L + \Delta R)$$

$$r_t = \frac{r(\Delta L + \Delta R)}{\Delta R - \Delta L}$$

Using trigonometry, we can find the movement in each direction.

$$\Delta x = r_t(\cos\Delta\theta - 1)$$

$$\Delta y = r_t \sin\Delta\theta$$

Strafing

The angle equation that we found for forwards/backwards movements works for strafing as well, so there is no need to create a new one. But we can solve for the strafing radius using the arc length formula.

$$\Delta B = (r_s + r_b)\Delta\theta$$

$$\Delta B = r_s\Delta\theta + r_b\Delta\theta$$

$$r_s\Delta\theta = \Delta B - r_b\Delta\theta$$

$$r_s = \frac{\Delta B}{\Delta\theta} - r_b$$

Using trigonometry, we can find the movement in each direction. It is important to note the rotated axes relative to the robot, as it's strafing.

$$\Delta x = r_s \sin\Delta\theta$$

$$\Delta y = r_s(1 - \cos\Delta\theta)$$

Final Equations

After deriving the equations for forwards/backwards as well as strafing movement, we must add the two components together to get the final equations.

$$\Delta x = r_t(\cos\Delta\theta - 1) + r_s \sin\Delta\theta$$

$$\Delta y = r_t \sin\Delta\theta + r_s(1 - \cos\Delta\theta)$$

As previously mentioned, because we treat all paths as arcs, straight lines must be interpreted as arcs with infinite radius. Although this can be made to work with our equations by using limits and L'Hôpital's Rule, we need to create separate line equations to implement in the code.

As these are lines, $\Delta\theta = 0$, which makes everything a lot easier. The change in x is the change in the back encoder, while the change in y is the average of the changes in the two forward encoders.

So then the final equations are:

$$\Delta\theta = \frac{\Delta R - \Delta L}{2r}$$

If $\Delta\theta = 0$:

$$\Delta x = \Delta B$$
$$\Delta y = \frac{\Delta L + \Delta R}{2}$$

And if $\Delta\theta \neq 0$:

$$r_t = \frac{r(\Delta L + \Delta R)}{\Delta R - \Delta L}$$
$$r_s = \frac{\Delta B}{\Delta\theta} - r_b$$
$$\Delta x = r_t(\cos\Delta\theta - 1) + r_s\sin\Delta\theta$$
$$\Delta y = r_t\sin\Delta\theta + r_s(1 - \cos\Delta\theta)$$